Criteria Used for Field Error Specifications

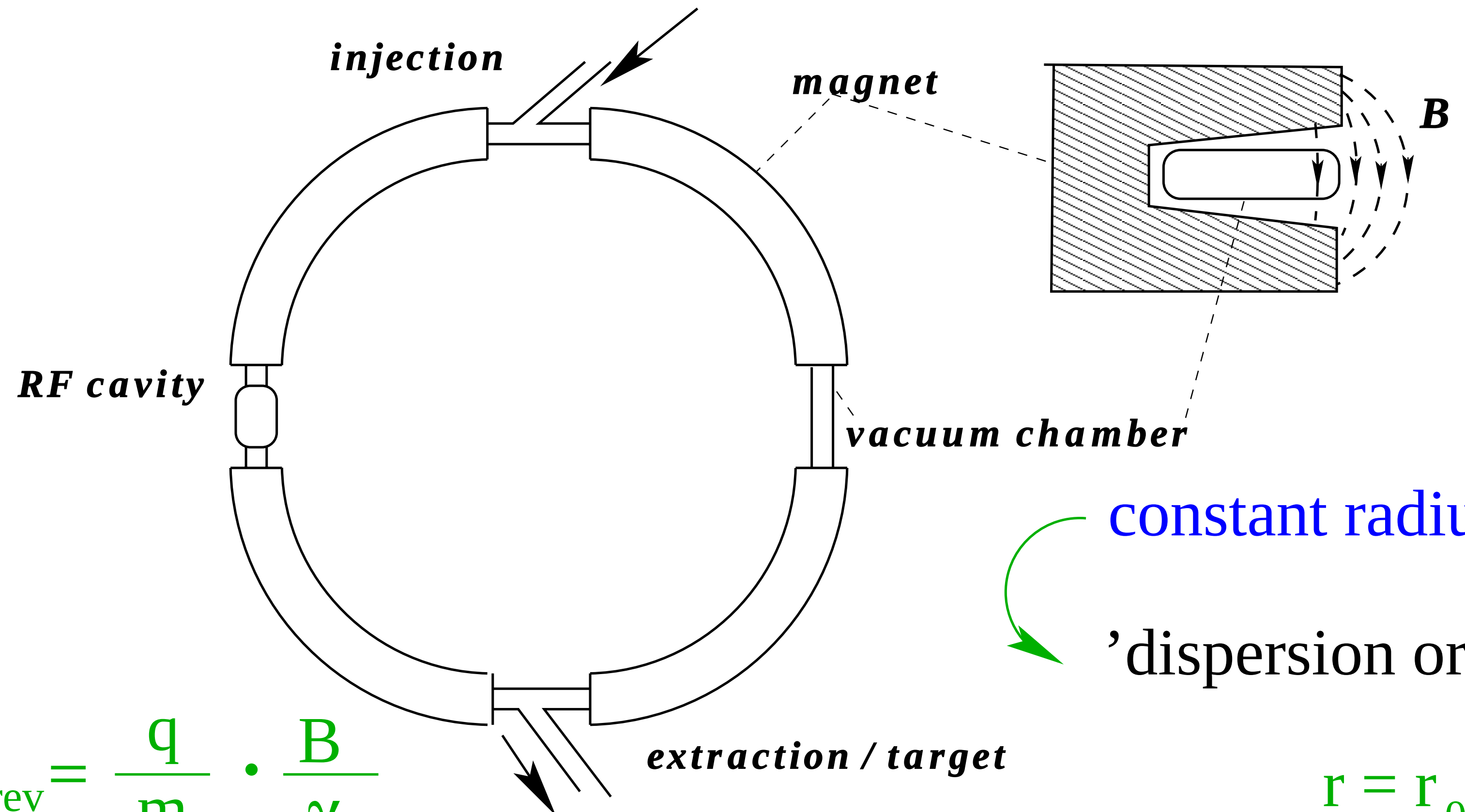
- *Particle Motion in a Synchrotron*
- *Non−linear Field Errors and Dynamic Aperture*
- *Specification Criteria*
- *Field Error Definitions*
- *Assumptions on Alignment Errors*
- *Specification Error Table in LHCPR 501*

Particle Motion in a Storage Ring

ideal particle motion in a synchrotron:

- revolution around the storage ring: $r = \frac{m_0}{q} \cdot \frac{\gamma}{B} \cdot v$

$v = c$

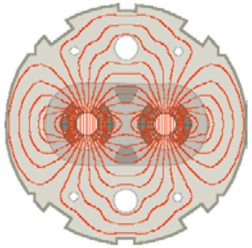


constant radius for constant $\frac{\gamma}{B}$

'dispersion orbit' for energy error

$$r = r_0 + D_x(s) \cdot \frac{\delta p}{p_0}$$

$$\omega_{rev} = \frac{q}{m_0} \cdot \frac{B}{\gamma}$$



Particle Motion in a Storage Ring

ideal particle motion:

transverse focusing via quadrupole magnets: example gravitation:

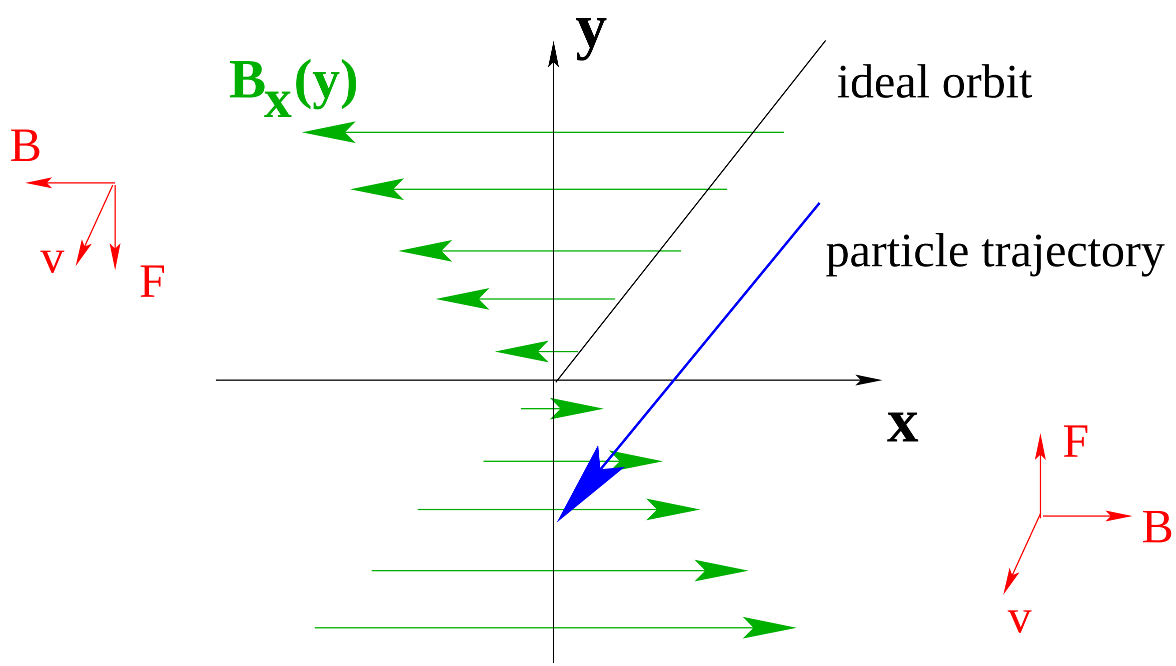
$g = 10 \bullet m \bullet s^{-2}$ $\quad \Delta s = \frac{1}{2} \bullet g \bullet \Delta t^{2}$ $\quad \Delta s = 18$ mm

→ $\Delta t = 60$ msec

= 660 Turns!

→

requires focusing!



Alternate Gradient Focusing

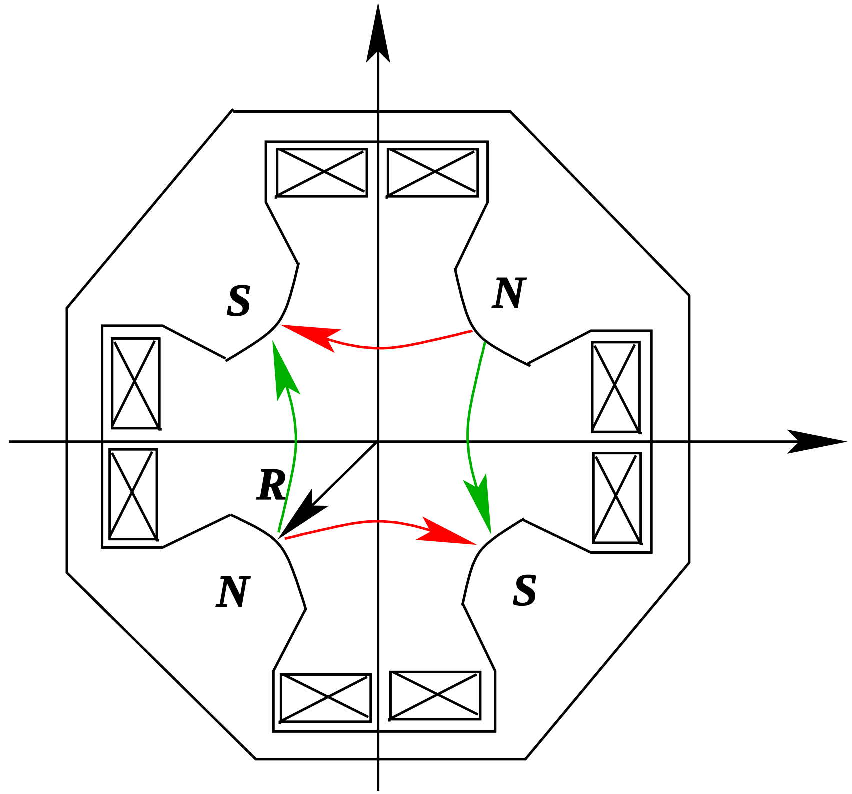
- Quadrupole Magnet

$$B_x = -g \cdot y$$

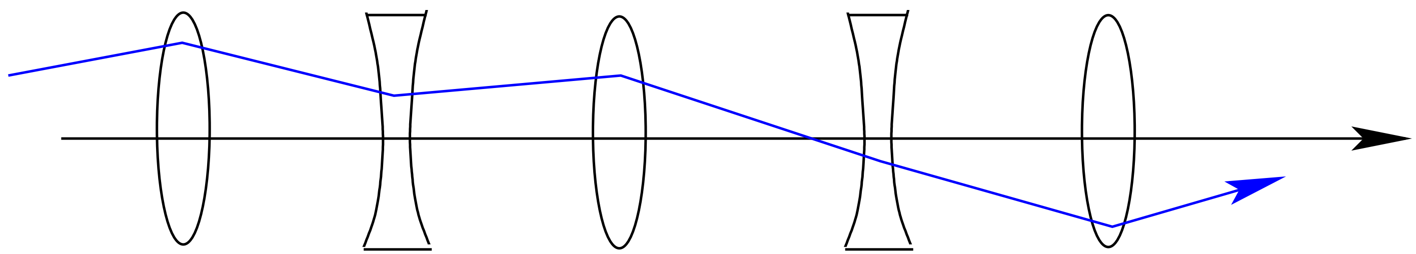
$$B_y = -g \cdot x$$

$$F_x = q \cdot v \cdot g \cdot x$$

$$F_y = -q \cdot v \cdot g \cdot y$$



- Alternate Gradient Focusing



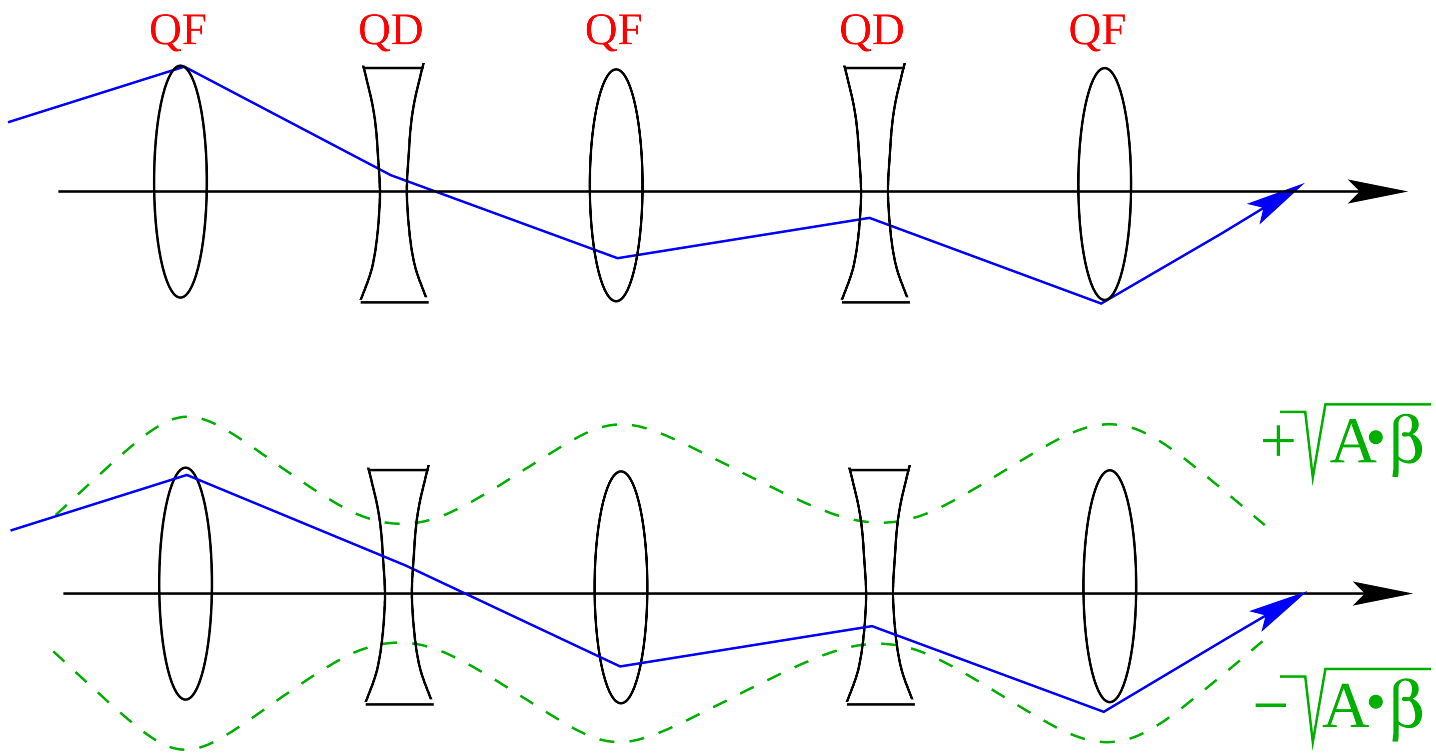
Idea: cut the arc sections in bending focusing and defocusing elements

Optic Functions

- individual particle oscillations: $x = \sqrt{A \cdot \beta(s)} \cdot \sin(\phi(s) + \phi_0)$

→ β, ϕ are determined by the quadrupole arrangement and powering

- envelope function:



Optic Functions

- particle tune:

  $Q_{x,\,y}$ = number of transverse oscillation in one revolution

  independent motion in the horizontal and vertical plane!

- envelope function:

  beam size depends on local β −function and injected particle density

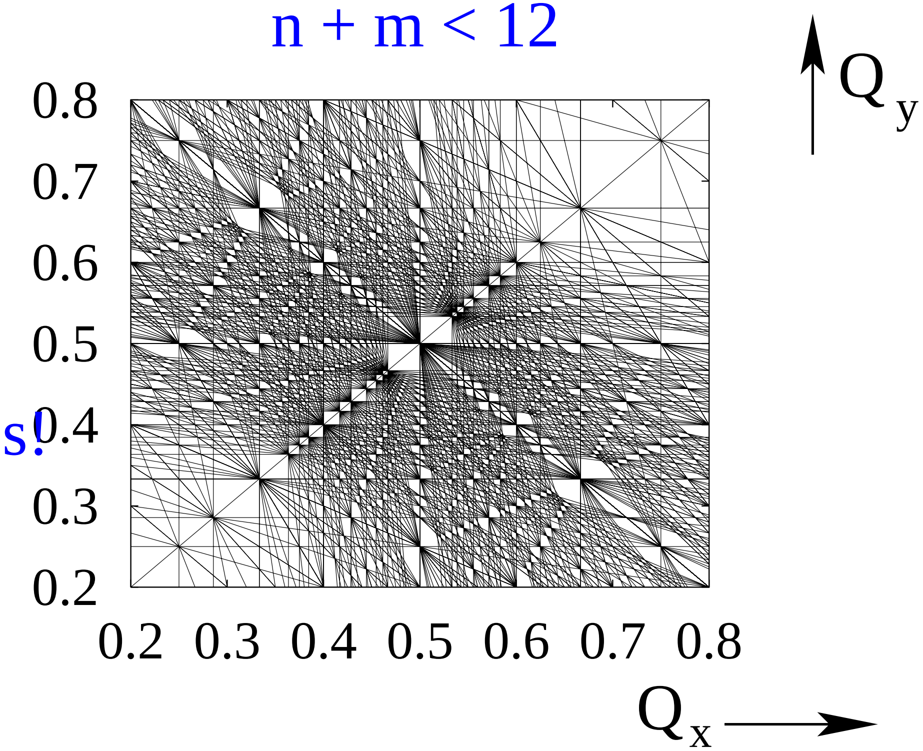
  $$\sigma_z = \sqrt{\varepsilon_z \cdot \beta_z(s)} \quad ; \quad z = x, y$$

- chromaticity: quadrupole focusing depends on the particle energy

  $$Q = Q_0 + Q' \cdot \frac{\delta p}{p_0} + Q'' \cdot \left(\frac{\delta p}{p_0}\right)^2 + \ldots.$$

Resonances and Non−Linear Field Errors

- resonances in the tune diagram:



$$n \cdot Q_x + m \cdot Q_y = r$$

magnetic field imperfections drive resonances!

→ particle loss

→ DA = maximum stable amplitude

- resonances limit the long term stability of the protons:

$$h_{n,m} \propto A^{n+m}$$ → avoid 'low order' resonances

- experience from Sp$\bar{\text{p}}$S, Tevatron and HERA: avoid resonances < 11 th order!

Resonances

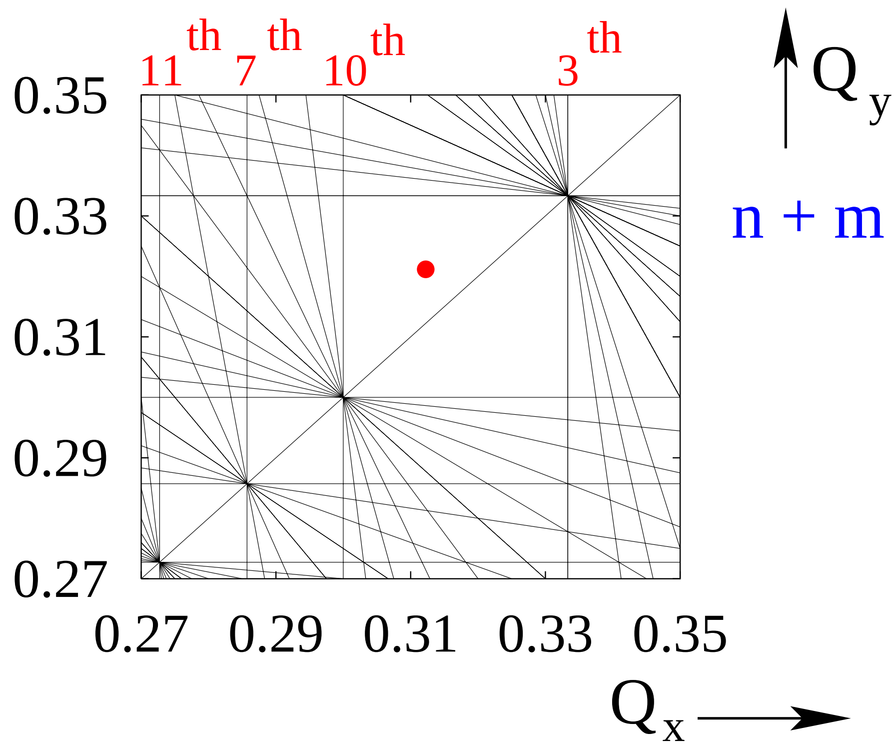
- detuning with amplitude:

  non−linear fields change Q as function of the oscillation amplitude

- LHC working point:

  $Q_x = 64.28;\quad Q_y = 59.31$

  → requires small coupling between horizontal and vertical motion



n + m < 12

→ total tune spread must be sufficiently small!

Multipole Error Feed-Down

- ***multipole error coefficients:***

$$B_y + i \cdot B_x = B_0 \cdot \sum_n (b_n + i \cdot a_n) \cdot \frac{(x + i \cdot y)^{n-1}}{R_{ref}^{n-1}}$$

17mm for the LHC

- ***multipole error feed down:***

$$\Delta b_n + i \Delta a_n = \frac{(k-1)!}{(n-1)!(k-n)!} \cdot (b_k + i\, a_k) \cdot \left(\frac{\Delta x}{R_{ref}} \right)^{k-n}$$

Feed−Down Errors

quadrupole alignment errors:

additional dipole deflections ⟶ particle motion does not follow the ideal reference orbit

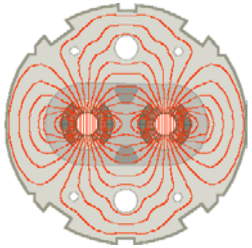
⟶ the magnets are installed with respect to the unperturbed reference orbit

⟶ feed−down errors in other field perturbations

sextupole alignment errors:

additional quadrupole focusing ⟶ tune depends on orbit

dispersion orbit in sextupole fields ⟶ $Q = Q_0 + Q' \cdot \frac{\delta p}{p_0} + Q'' \cdot \left(\frac{\delta p}{p_0}\right)^2 + ..$



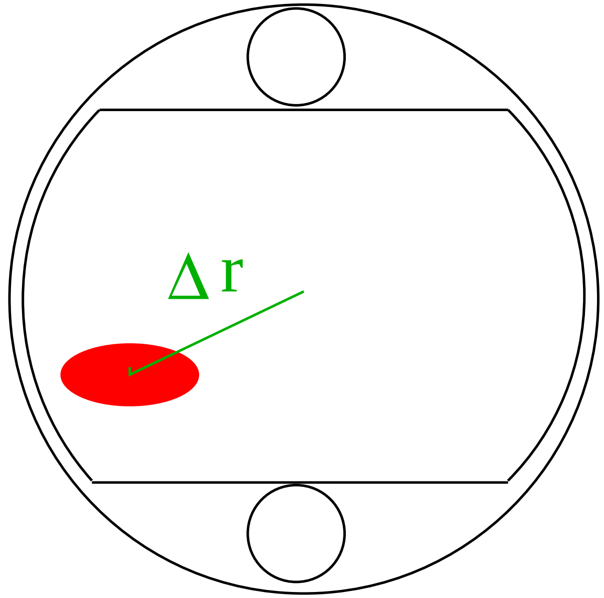
Effect of the Magnet Errors on the Beam

- b_1; a_1: *orbit perturbation*
- b_2: *β−beat*
- a_2: *linear coupling:* ($c\_ = 0.3$)
- b_3: *chromaticity* ($Q' = 500$ units)
- a_3: *chromatic coupling:* $a_2 = a_3 \cdot \delta$; Q''
- b_4: *detuning with amplitude;* Q''; (2, −2) *resonance*
- a_4: (1, −1) *resonance*
- b_5: Q'''; *chromatic detuning with amplitude:* $\frac{\partial^2 Q}{\partial J \partial \delta}$
- $b_n + a_n$: *long term stability of particle motion* DA

Specification Criteria

mechanical aperture:

the orbit excursion plus 10 times the rms beam size must fit into the mechanical aperture



magnet quench → hard limits!

- beam offset depends on closed orbit, dispersion and energy error

  → imposes limits on CO, D, and $\Delta p/p_0$

- the beam size depends on β −function, dispersion and energy spread

  → imposes limits on β −beat, D, and $\delta p/p_0$

Specification Criteria

mechanical aperture:

- $\epsilon_n = 3.75 \cdot 10^{-6}$ m and $\beta_{max} = 180$m → σ_{max}=1.2mm at injection
- collimator jaws at 7σ and 8.2σ

→
- β −beat < 21% (25%)
- closed orbit (horizontal and vertical) < 4mm (3mm at 7TeV)
- parasitic dispersion: $\Delta D_{x,y} < \sqrt{\frac{\beta_{x,y}}{\beta_{F,QF}}} \cdot D_{x,QF} \cdot 0.3 \ (0.28)$
- momentum spread: $\frac{\Delta p}{p_0} < +/-1.0 \cdot 10^{-3} \ (0.36 \cdot 10^{-3})$
- momentum deviation: $\frac{\Delta p}{p_0} < +/-2.0 \cdot 10^{-3} \ (0.5 \cdot 10^{-3})$

Specification Criteria

resonances and beam stability:

- $\Delta Q < +/- 5.0 \cdot 10^{-3}$ and $0 < \Delta Q' < 2$:
  - $Q'' = +/- 10^{3} \quad (+/- 2.0 \cdot 10^{3})$
  - $Q''' = -0.5 \cdot 10^{6} <-> +3.0 \cdot 10^{6} \quad (-4.0 \cdot 10^{6} <-> +4.0 \cdot 10^{6})$
  - $\frac{\partial Q}{\partial \varepsilon} = +/- 7.0 \cdot 10^{3}\ \mathrm{m}^{-1} \quad (+/- 8.0 \cdot 10^{3}\ \mathrm{m}^{-1}) \quad (\Delta Q = 2 \cdot 10^{-3} \text{ at } 6\sigma)$
  - $\frac{\partial^2 Q}{\partial \varepsilon \partial p} = +/- 7.0 \cdot 10^{6}\ \mathrm{m}^{-1} \quad (+/- 15.0 \cdot 10^{6}\ \mathrm{m}^{-1}) \quad (\Delta Q = 2 \cdot 10^{-3} \text{ at } 6\sigma)$ $(\delta_p = \delta_{max-bucket})$

Specification Criteria

- 100000 turn dynamic aperture without linear imperfections:
  - DA > 11.5 σ for minimum DA (resolution of +/− 0.5)

    experience shows that $DA_{machine}(20\ \text{min}) = \frac{DA_{non-linear}(100000\ \text{turns})}{2}$

  - the specification of MB field errors via the DA is based on:

    → keep the error table 9901 as a reference

    - select the field component to be studied and increase its reference value until the effect on the DA is more then 0.5σ

    - concerning the field errors for which a dedicated correction circuit exists scale the corrector circuit but allow for +/− 10% +/− 20% correction error

Specification Criteria

- ***corrector strength:***

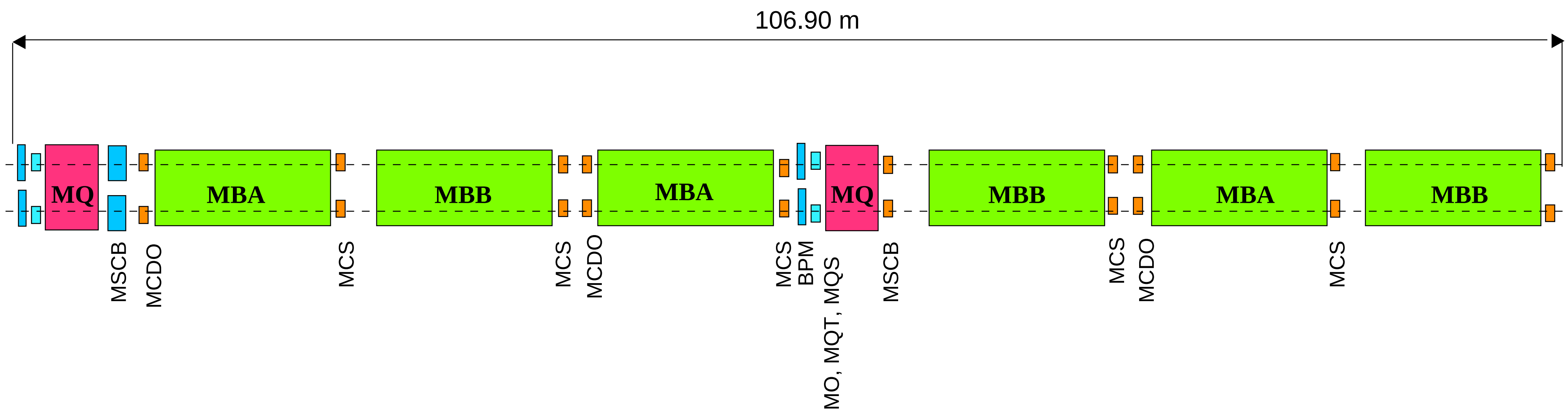
  - assure sufficient corrector strength for all LHC operation modes:

→

- injection optics at 7TeV
- collision optics at 7TeV
  assuming different combinations and phase advances between different sets of high luminosity insertions
- ultimate performance with 2 insertions with $\beta^* = 0.25$ meter

Correction Circuits

Schematic layout of one LHC cell (23 periods per arc)



MQT: trim quadrupole

MQS: skew trim quadrupole

MO: lattice octupole

MSCB: sextupole (skew sextupole) + orbit corrector

MCS: spool piece sextupole

MCDO: spool piece octupole + decapole

Alignment Errors

feed down of multipole errors I:

SSS:

feed down errors must be smaller then main dipole field errors

(β−beat due to orbit errors in the SSS sextupoles is equal to the β−beat due random b_2 errors of the main dipole magnets)

→

- rms closed orbit = 0.4mm [0.5mm/0.25mm (QF/QD)]
- 0.5mrad rms roll error of main quadrupole magnets
- 0.43mm rms alignment error of lattice sextupole magnets

Alignment Errors

feed down of multipole errors II:

- spool piece alignment:

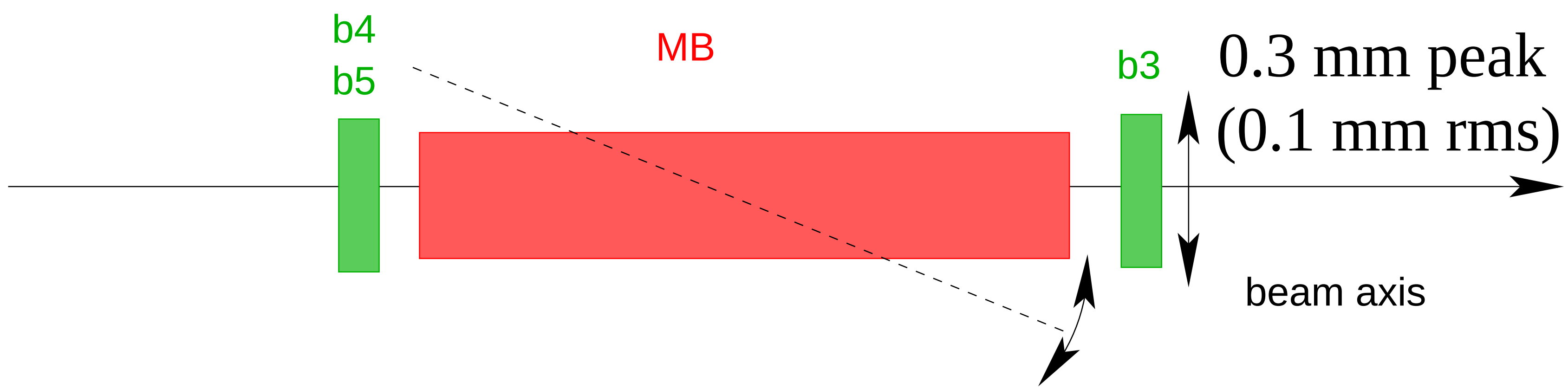
  feed down errors must be smaller then main dipole field errors

  β−beat due to sextupole spool piece alignment errors is equal to the β−beat due random b_2 errors of the main dipole magnets)

  watch out for feed down errors without corrector elements!

→
- 0.5mm rms tolerance for random alignment errors
- 0.1mm tolerance for systematic alignment errors [resolution of alignment measurements]

MB Alignment



→ spool piece alignment error has contributions from MB pitch

→ alignment strategy for magnets in LHC tunnel?

→ field quality specifications depend on the alignment tolerances!

Summary

- hard limits on the dipole field quality:

  - b_1, a_1: limited by available corrector strength

    → balance between MB and MQ contributions

  - b_2: limited by β − beat

    → MB specification depends on other sources for β−beat

  - a_2: limited by available corrector strength

  - b_3, a_3: limited by available corrector strength at top energy

Summary

'soft' limits on the dipole field quality:

- b_4, a_4: limited by acceptable tune spread and Dynamic Aperture

  → no correction circuit for a_4 field errors!

- b_5 : limited by non−linear chromaticity and Dynamic Aperture

- b_7 : limited by Dynamic Aperture

- b_n , a_n : limited by Dynamic Aperture

Multipole Error for 9901: Definitions

- ***types of errors:***
  - ***'systematic':*** average error over all magnets
  - ***'uncertainty':*** average error over one arc
  - ***'random':*** random component (truncated Gaussian / 3σ)

- ***error sources:***
  - ***'geometry':*** magnet geometry + saturation + screen
  - ***'persistent':*** persistent currents in the magnet cable
  - ***'ramp':*** ramp induced errors

Total Multipole Error

- total multipole error in the magnets:

$$a2_{tot} = a2gM + a2pM + a2tM + \frac{\xi_1}{1.5} \bullet \sqrt{a2gU^2 + a2pU^2 + a2tU^2}$$

$$+ \quad \xi_2 \bullet \sqrt{a2gR^2 + a2pR^2 + a2tR^2}$$

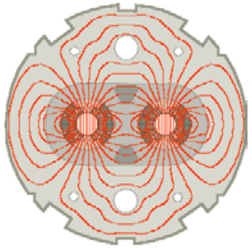
→ mixing of magnets from different manufacturers per octant

→ the concept of uncertainty is no longer fully applicable

- specifications must define limits for both random and systematic errors

- persistent current field error decay:

  dynamic effects of the field errors impose additional constraints on the machine operation!



Summary

- detailed field error specification depends on sharing between different error sources
- slightly different specification criteria for injection and collision optics
- slightly different specification criteria for systematic and random errors
- specification of the allowed range for systematic and uncertainty

→ detailed specification in LHC Project Report 501

→ presentation by Stephane Fartoukh

Harmonics a_n & b_n	Injection optics (450 GeV)	Injection optics (end of ramp)	Collision optics (7 TeV)	Systematic (max. value)	Uncertainty (max. value)	Random (r.m.s)	Criteria used
b_1	$\times$	$\times$	$\times$	None	6.5	8.0	Closed orbit and MCB strength at 7 TeV
a_1 (including dipole roll)	$\times$	$\times$	$\times$	6.5 (averaged per arc cell)		8.0	
b_2	$\times$		$\times$	1.4	0.8	0.7 0.8	β-beating and IP phasing
a_2	$\times$	$\times$	$\times$		0.9	1.9 2.3 1.6	Vertical dispersion, linear coupling and MQS strength at 7 TeV
b_3	$\times$		$\times$	10.7 3.0 (including the bias due to uncertainty)		1.4 1.8	b_2 feed-down at injection, off-momentum β-beating, MCS strength at 7 TeV
a_3	$\times$		$\times$		1.5	0.7	Chromatic coupling inducing Q'' and MSS strength at 7 TeV
b_4	$\times$		$\times$	± 0.2 (from Table 9901)	0.4	0.5	DA and Q'' at injection, MCO strength at 7 TeV
a_4	$\times$				0.2	0.5 (from Table 9901)	DA at injection
b_5	$\times$		$\times$	1.1 0.8 (including the bias due to uncertainty)		0.5 0.4	DA and Q''' at injection, MCD strength at 7 TeV
a_5	$\times$				0.4	0.4 (from Table 9901)	Off-momentum DA at injection
b_7	$\times$			$-0.3 < \langle b_7 \rangle < 0.1$		0.2 (from Table 9901)	DA at injection
a_6, b_6, a_7 and higher order multipoles	$\times$			OK with the Error Table 9901			DA at injection

Table 15: *Specifications for the dipole field quality at injection, end of ramp and in collision (a_n and b_n given in units of 10^{-4} relative field error at a reference radius $R_{ref} = 17$ mm).*