

## D1 Cold Transfer Function: HCMBX 001-BL00000x

Five of these single aperture magnets were produced and they will be used in the D1 position next to ALICE and LHCb. Only two magnets were measured cold, D1L101 and D1L103. However, D1L101 is not completely representative since it contains stainless steel yoke keys instead of steel.

First of all, define some "constants"

$$I_{inj} := 400 \quad I_{nom} := 6000 \quad I_c := 7000$$

The geometric term is estimated to be

$$\gamma := 0.00668126$$

The TF data was measured at 4.5K

data :=

	0	1
0	50.85	$6.756 \cdot 10^{-3}$
1	100.834	$6.668 \cdot 10^{-3}$
2	200.615	$6.658 \cdot 10^{-3}$
3	300.565	$6.665 \cdot 10^{-3}$
4	350.504	$6.668 \cdot 10^{-3}$
5	400.493	$6.67 \cdot 10^{-3}$
6	600.231	$6.675 \cdot 10^{-3}$
7	799.804	...

$$\text{current} := \text{data}^{(0)}$$

$$\text{TF} := \text{data}^{(1)}$$

$$\text{temp1} := \text{TF} - \gamma$$

$$\text{dataA} := \text{augment}(\text{current}, \text{temp1}, \text{TF})$$

### DC Component - Saturation

Prepare the data to fit between 2000A and  $I_{nom}$ . Scale the TF by  $10^5$  to improve conditioning

$$\text{saturation}^{(0)} := \text{submatrix}(\text{dataA}, 13, 33, 0, 0)$$

$$\text{saturation}^{(1)} := \text{submatrix}(\text{dataA}, 13, 33, 1, 1) \cdot 100000$$

$$Xs := \text{saturation}^{(0)}$$

$$Ys := \text{saturation}^{(1)}$$

temp1 =

	0
0	$7.483 \cdot 10^{-5}$
1	$-1.366 \cdot 10^{-5}$
2	$-2.336 \cdot 10^{-5}$
3	$-1.609 \cdot 10^{-5}$
4	$-1.35 \cdot 10^{-5}$
5	$-1.129 \cdot 10^{-5}$
6	$-5.978 \cdot 10^{-6}$
7	$-3.749 \cdot 10^{-6}$
8	$-2.91 \cdot 10^{-6}$
9	$-2.756 \cdot 10^{-6}$
10	$-2.898 \cdot 10^{-6}$
11	$-3.381 \cdot 10^{-6}$
12	$-4.046 \cdot 10^{-6}$
13	$-4.866 \cdot 10^{-6}$
14	$-5.983 \cdot 10^{-6}$
15	...

dataA =

	0	1	2
0	50.85	$7.483 \cdot 10^{-5}$	$6.756 \cdot 10^{-3}$
1	100.834	$-1.366 \cdot 10^{-5}$	$6.668 \cdot 10^{-3}$
2	200.615	$-2.336 \cdot 10^{-5}$	$6.658 \cdot 10^{-3}$
3	300.565	$-1.609 \cdot 10^{-5}$	$6.665 \cdot 10^{-3}$
4	350.504	$-1.35 \cdot 10^{-5}$	$6.668 \cdot 10^{-3}$
5	400.493	$-1.129 \cdot 10^{-5}$	$6.67 \cdot 10^{-3}$
6	600.231	$-5.978 \cdot 10^{-6}$	$6.675 \cdot 10^{-3}$
7	799.804	$-3.749 \cdot 10^{-6}$	$6.678 \cdot 10^{-3}$
8	999.853	$-2.91 \cdot 10^{-6}$	$6.678 \cdot 10^{-3}$
9	$1.2 \cdot 10^3$	$-2.756 \cdot 10^{-6}$	$6.679 \cdot 10^{-3}$
10	$1.4 \cdot 10^3$	$-2.898 \cdot 10^{-6}$	$6.678 \cdot 10^{-3}$
11	$1.6 \cdot 10^3$	$-3.381 \cdot 10^{-6}$	$6.678 \cdot 10^{-3}$
12	$1.8 \cdot 10^3$	$-4.046 \cdot 10^{-6}$	$6.677 \cdot 10^{-3}$
13	$2 \cdot 10^3$	$-4.866 \cdot 10^{-6}$	$6.676 \cdot 10^{-3}$
14	$2.201 \cdot 10^3$	$-5.983 \cdot 10^{-6}$	$6.675 \cdot 10^{-3}$
15	$2.401 \cdot 10^3$	$-7.544 \cdot 10^{-6}$	...

Define the saturation function model (EDMS 908232)

$$f_{\text{sat}}(I, \sigma, S, I_0) := \begin{cases} \sigma \cdot \left( 1 - 0.5 \cdot \operatorname{erfc} \left( S \cdot \frac{|I - I_0|}{I_{\text{nom}}} \right) \right) & \text{if } (I - I_0) > 0 \\ \sigma \cdot \left( 0.5 \cdot \operatorname{erfc} \left( S \cdot \frac{|I - I_0|}{I_{\text{nom}}} \right) \right) & \text{otherwise} \end{cases}$$

Define a vector of guesses for the fit.

$$\text{guess1} := \begin{pmatrix} 0 \\ 0 \\ 5000 \end{pmatrix}$$

Use **genfit** to find the parameters in the model function.

$$\text{sat} := \text{genfit}(Xs, Ys, \text{guess1}, f_{\text{sat}})$$

Here are the fitted coefficients for the saturation.

$$\text{sat} = \begin{pmatrix} -58.333 \\ 3.417 \\ 5.176 \times 10^3 \end{pmatrix}$$

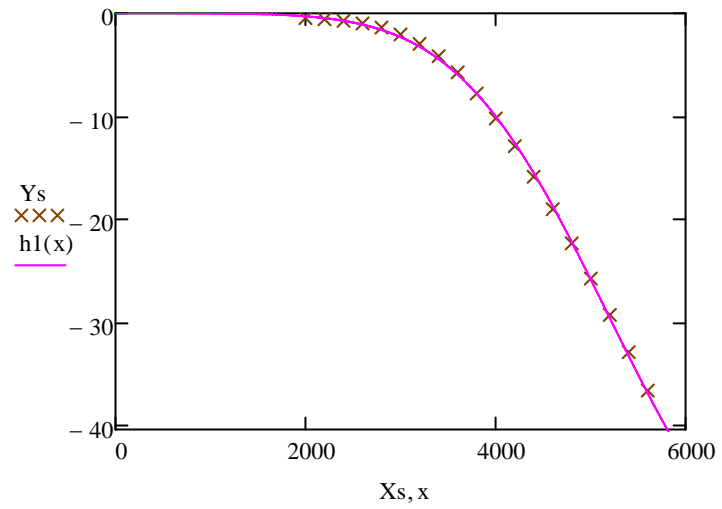
saturation =

	0	1
0	$2 \cdot 10^3$	-0.487
1	$2.201 \cdot 10^3$	-0.598
2	$2.401 \cdot 10^3$	-0.754
3	$2.601 \cdot 10^3$	-0.996
4	$2.801 \cdot 10^3$	-1.407
5	$3.002 \cdot 10^3$	-2.067
6	$3.202 \cdot 10^3$	-2.986
7	$3.402 \cdot 10^3$	-4.19
8	$3.602 \cdot 10^3$	-5.764
9	$3.802 \cdot 10^3$	-7.823
10	$4.002 \cdot 10^3$	-10.25
11	$4.202 \cdot 10^3$	-12.952
12	$4.402 \cdot 10^3$	-15.912
13	$4.602 \cdot 10^3$	-19.083
14	$4.803 \cdot 10^3$	-22.396
15	$5.003 \cdot 10^3$	...

Define a function using these coefficients

$$h1(x) := \begin{cases} \text{sat}_0 \cdot \left( 1 - 0.5 \cdot \text{erfc} \left( \text{sat}_1 \cdot \frac{|x - \text{sat}_2|}{\text{Inom}} \right) \right) & \text{if } (x - \text{sat}_2) > 0 \\ \text{sat}_0 \cdot \left( 0.5 \cdot \text{erfc} \left( \text{sat}_1 \cdot \frac{|x - \text{sat}_2|}{\text{Inom}} \right) \right) & \text{otherwise} \end{cases}$$

and plot the fitted function together with the original data (minus the geometric)



## DC Component - Persistent Currents

Prepare the data to fit between 200A and Inom. Fit against the TF-geom-saturation.  
Remember to scale the saturation function by  $10^{-5}$  !!

$$\text{pers}^{(0)} := \text{submatrix}(\text{dataA}, 2, 33, 0, 0)$$

$$Xp := \text{pers}^{(0)}$$

$$\text{pers}^{(1)} := \text{submatrix}(\text{dataA}, 2, 33, 1, 1) - \text{h1}(Xp) \cdot 0.00001$$

$$n := \text{length}(Xp) - 1$$

$$Yp := \text{pers}^{(1)}$$

Initial guess for parameters:

$$\mu := 0 \quad p := 0 \quad q := 0$$

$$\text{guessp} := \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$f_{\text{per}}(I, \mu, p, q) := \mu \cdot \left( \frac{I_{\text{inj}}}{I} \right)^{2-p} \left( \frac{I_c - I}{I_c - I_{\text{inj}}} \right)^q$$

	0	1
0	200.615	-2.335 · 10 <sup>-5</sup>
1	300.565	-1.606 · 10 <sup>-5</sup>
2	350.504	-1.347 · 10 <sup>-5</sup>
3	400.493	-1.125 · 10 <sup>-5</sup>
4	600.231	-5.911 · 10 <sup>-6</sup>
5	799.804	-3.625 · 10 <sup>-6</sup>
6	999.853	-2.685 · 10 <sup>-6</sup>
7	1.2 · 10 <sup>3</sup>	-2.358 · 10 <sup>-6</sup>
8	1.4 · 10 <sup>3</sup>	-2.21 · 10 <sup>-6</sup>
9	1.6 · 10 <sup>3</sup>	-2.22 · 10 <sup>-6</sup>
10	1.8 · 10 <sup>3</sup>	-2.133 · 10 <sup>-6</sup>
11	2 · 10 <sup>3</sup>	-1.789 · 10 <sup>-6</sup>
12	2.201 · 10 <sup>3</sup>	-1.149 · 10 <sup>-6</sup>
13	2.401 · 10 <sup>3</sup>	-1.254 · 10 <sup>-7</sup>
14	2.601 · 10 <sup>3</sup>	1.157 · 10 <sup>-6</sup>
15	2.801 · 10 <sup>3</sup>	...

pers =

## Levenberg-Marquardt

Use the Levenberg-Marquardt method to minimize this problem. The Levenberg-Marquardt method does its own summing and squaring of the residuals, so the equation to minimize in the **Minerr** Solve Block is

$$\text{resid}(\mu, p, q) := Y_p - \overrightarrow{f_{per}(X_p, \mu, p, q)}$$

Given

$$\begin{aligned} 0 &= \text{resid}(\mu, p, q) \\ 2 &\geq q \end{aligned}$$

Parameters for best fit:

$$\begin{pmatrix} \mu 1 \\ p 1 \\ q 1 \end{pmatrix} := \text{Minerr}(\mu, p, q)$$

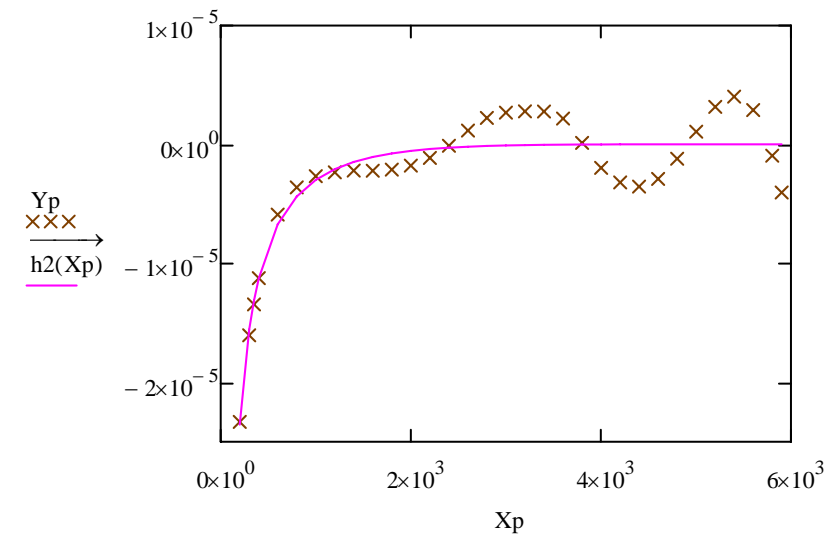
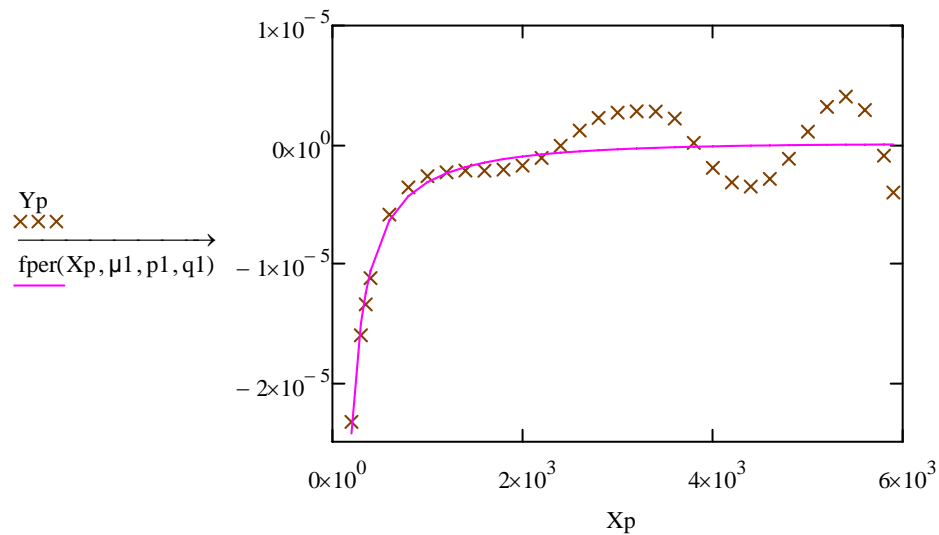
$$\begin{pmatrix} \mu 1 \\ p 1 \\ q 1 \end{pmatrix} = \begin{pmatrix} -1.065 \times 10^{-5} \\ 0.89 \\ 2 \end{pmatrix}$$

$$\text{per} := \text{genfit}(X_p, Y_p, \text{guessp}, f_{per}) \quad \text{per} = \begin{pmatrix} -1.129 \times 10^{-5} \\ 1.204 \\ 6.28 \end{pmatrix}$$

Sum of squares implicitly minimized by this method:

$$\text{SSE}(\mu, p, q) := \sum \text{resid}(\mu, p, q)^2$$

$$h_2(y) := \text{per}_0 \left( \frac{\text{linj}}{y} \right)^{2-\text{per}_1} \left( \frac{\text{Ic} - y}{\text{Ic} - \text{linj}} \right)^{\text{per}_2}$$



Mean squared error (zero if a true solution existed):

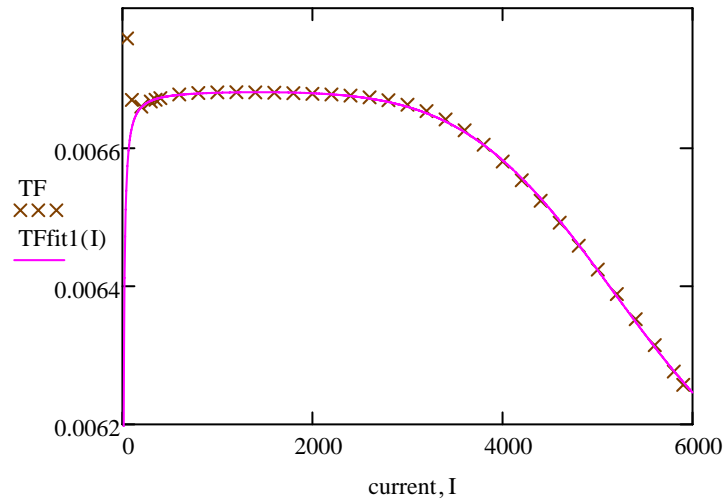
$$\frac{\text{SSE}(\mu^1, p^1, q^1)}{n - 2} = 4.783 \times 10^{-12}$$

You can see that this is the same number that is minimized by the solver by checking the internal error variable, **ERR**. In the case of Levenberg-Marquardt, ERR represents the square root of the SSE (Sum Squared Error).

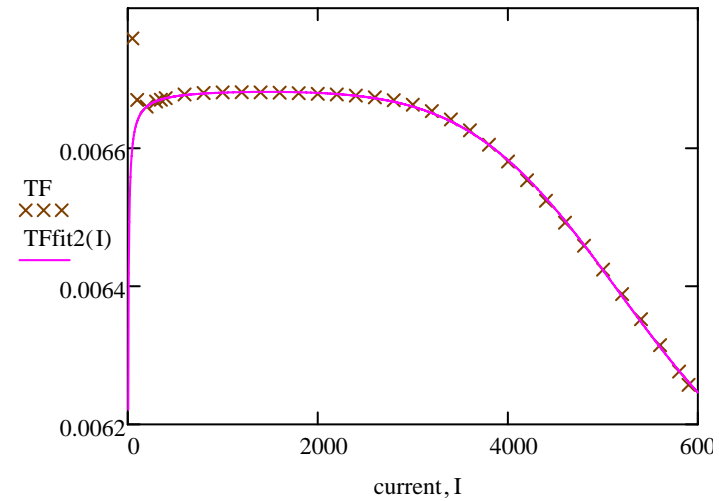
$$\left( \frac{\text{ERR}^2}{n - 2} \right) = 4.782637 \times 10^{-12}$$

Compute the complete fitted transfer function

$$\text{TFfit1}(I) := 0.00001 h_1(I) + \text{fper}(I, \mu_1, p_1, q_1) + \gamma$$



$$\text{TFfit2}(I) := 0.00001 h_1(I) + h_2(I) + \gamma$$



Evaluate the relative field error due to the fit (units)

$$\text{TFerror1} := \frac{(\overrightarrow{\text{TFfit1}(\text{current})} - \text{TF}) \cdot 10000}{0.5 \cdot (\overrightarrow{\text{TFfit1}(\text{current})} + \text{TF})}$$

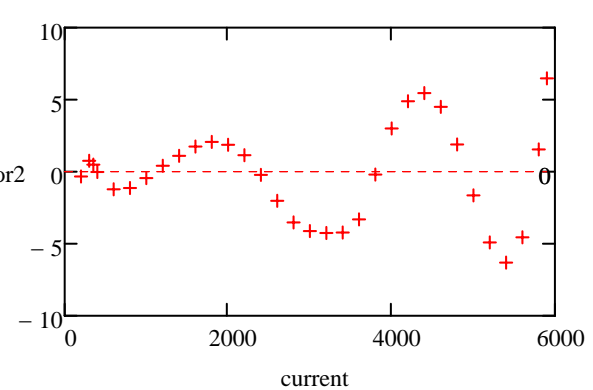
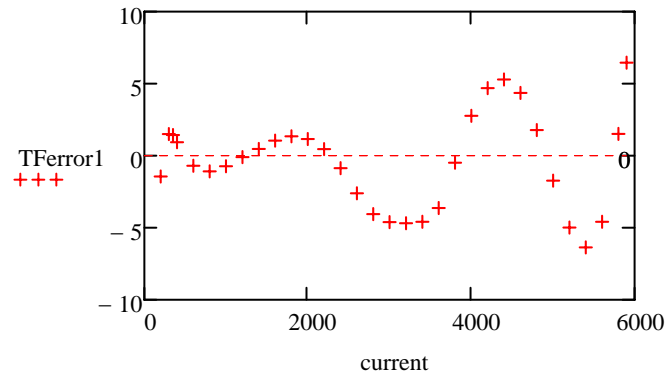
TFerror1 =

	0
0	-287.317
1	-60.288
2	-1.458
3	1.49
4	1.422
5	0.924
6	-0.706
7	-1.095
8	-0.747
9	-0.109
10	0.45
11	1.032
12	1.33
13	1.146
14	0.449
15	...

$$\text{TFerror2} := \frac{(\overrightarrow{\text{TFfit2}(\text{current})} - \text{TF}) \cdot 10000}{0.5 \cdot (\overrightarrow{\text{TFfit2}(\text{current})} + \text{TF})}$$

TFerror2 =

	0
0	-232.78
1	-46.647
2	-0.347
3	0.74
4	0.482
5	-0.035
6	-1.235
7	-1.15
8	-0.461
9	0.398
10	1.086
11	1.734
12	2.052
13	1.859
14	1.132
15	...



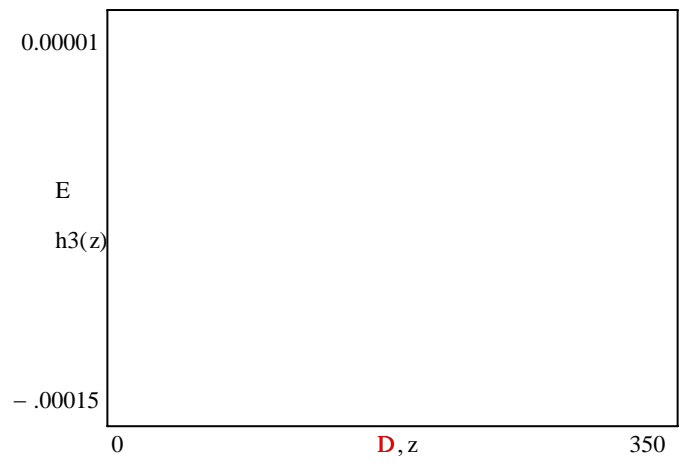
$$\text{fres}(I, \rho, r, \text{offs}) := \rho \cdot \left( \frac{\text{linj}}{I} \right)^r + \text{offs}$$

$$\text{guessr} := \begin{pmatrix} 0 \\ 0 \\ -0.0001 \end{pmatrix}$$

$$\text{res} := \text{genfit}(\mathbf{D}, E, \text{guessr}, \text{fres})$$

$$h3(z) := \text{res}_0 \cdot \left( \frac{\text{linj}}{z} \right)^{\text{res}_1} + \text{res}_2$$







	0
0	$-2.358 \cdot 10^{-5}$
1	$-1.557 \cdot 10^{-5}$
2	$-1.314 \cdot 10^{-5}$
3	$-1.127 \cdot 10^{-5}$
4	$-6.735 \cdot 10^{-6}$
5	$-4.393 \cdot 10^{-6}$
6	$-2.993 \cdot 10^{-6}$
7	$-2.092 \cdot 10^{-6}$
8	$-1.485 \cdot 10^{-6}$
9	$-1.062 \cdot 10^{-6}$
10	$-7.627 \cdot 10^{-7}$
11	$-5.482 \cdot 10^{-7}$
12	$-3.93 \cdot 10^{-7}$
13	$-2.806 \cdot 10^{-7}$
14	$-1.99 \cdot 10^{-7}$
15	...

$\xrightarrow{\quad}$   
 $h_2(X_p) =$